



MATHEMATICS SPECIALIST Year 12

Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading Time:	4 minutes
Working time for this section:	40 minutes
Marks available:	41 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable/gel pens
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Please turn over the find Question 1.

See next page

Question 1**(5 marks)**

Let $z = x + yi$ be a complex number where $x > 0, y > 0$. Let $w = iz - \bar{z}$.

Determine $|w|$ in terms of x and/or y and determine the value of $\arg(w)$.

Question 2**(4 marks)**

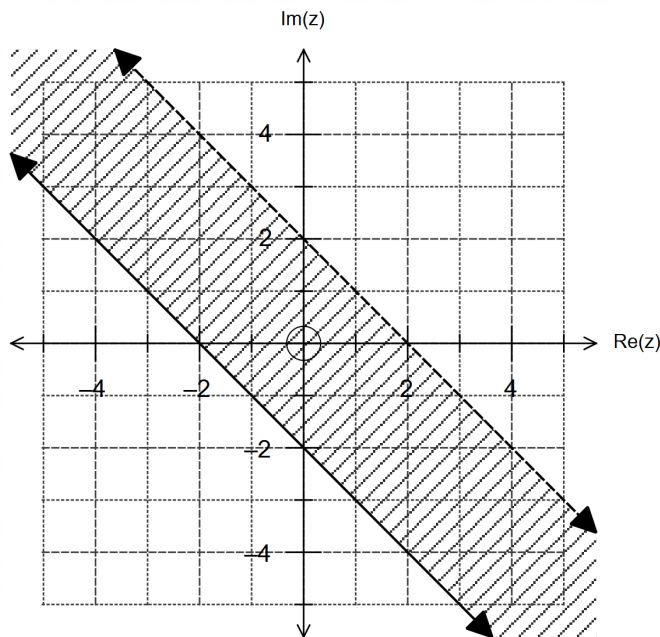
- (a) Given that $a = 3 \left(\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right)$ and $b = 4 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right)$, show that $\frac{a}{b}$ is purely imaginary. (2 marks)

- (b) Express $8 \left(\cos \left(\frac{\pi}{3} \right) - i \sin \left(\frac{\pi}{3} \right) \right)$ in Cartesian form. (2 marks)

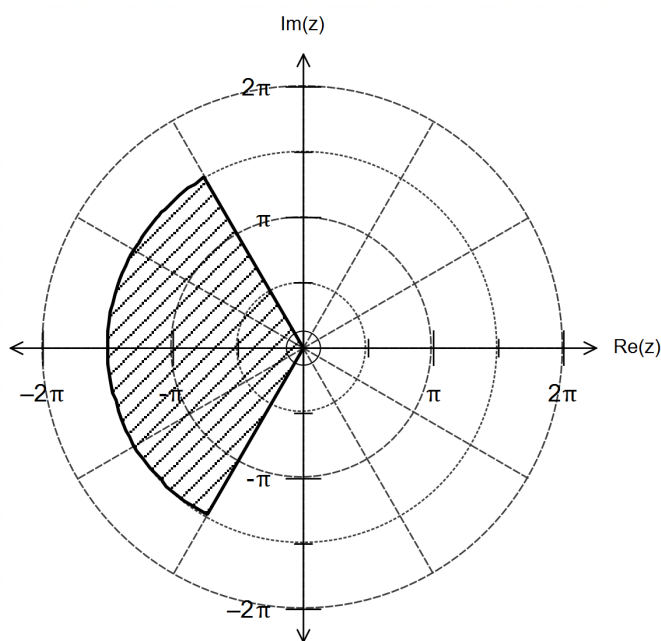
Question 3

(10 marks)

- (a) State the conditions on the complex number $z = a + bi$ that describe the region given below. (3 marks)

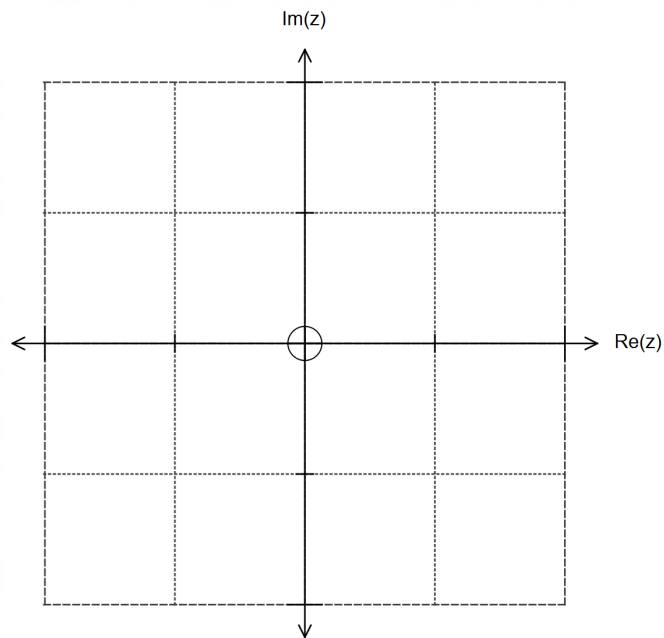


- (b) State the conditions on the complex number $z = r \operatorname{cis} \theta$ that describe the region given below, where $r \geq 0$ and $-\pi < \theta \leq \pi$. (3 marks)



Question 3 continued

- (c) Sketch the following set of complex numbers z in the Argand plane that satisfy the condition: $z^2 + \bar{z}^2 = 0$. (4 marks)



Question 4**(9 marks)**

Consider the complex number $z = \cos \theta + i \sin \theta$.

- (a) Rationalise $\frac{1}{z}$ to show that $z^{-1} = \cos \theta - i \sin \theta$. (2 marks)

- (b) It can be shown that $z^n = \cos(n\theta) + i \sin(n\theta)$ and $z^{-n} = \cos(n\theta) - i \sin(n\theta)$.

Use this information to show that $\sin(n\theta) = \frac{z^n - z^{-n}}{2i}$ and $\cos(n\theta) = \frac{z^n + z^{-n}}{2}$. (2 marks)

Question 4 continued

- (c) Using your answer to (b), write an expression for $\cos(2\theta)$ and $\sin(4\theta)$ in terms of z . (2 marks)

- (d) Using your answer to (c), determine an expression for $\cos(2\theta) \times \sin(4\theta)$ in terms of $\sin(6\theta)$ and $\sin(2\theta)$. (3 marks)

Question 5**(13 marks)**

(a) The function $f(z) = 2z^3 - z^2 + 6z - 3$ is defined for $z \in \mathbb{C}$.

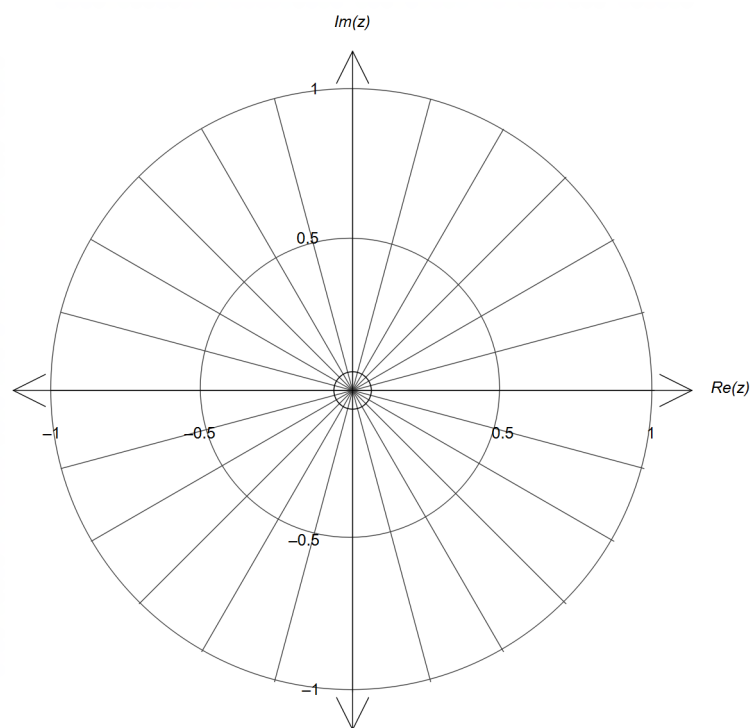
(i) Show that $(z + \sqrt{3}i)$ is a factor of $f(z)$. (2 marks)

(ii) Given that $(z + \sqrt{3}i)$ is a factor of $f(z)$, state another factor of $f(z)$. (1 mark)

(iii) Hence, or otherwise, solve the equation $2z^3 + 6z = z^2 + 3$ (4 marks)

Question 5 continued

- (b) Determine all the solutions to the equation $z^3 + 2^{-3} = 0$ in the form $z = a + bi$, and then sketch all the solutions on the grid provided below. (6 marks)



End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____